Exercise 92

The cost, in dollars, of producing x units of a certain commodity is

$$C(x) = 920 + 2x - 0.02x^2 + 0.00007x^3$$

(a) Find the marginal cost function.

(b) Find $C'(100)$ and explain its meaning.

(c) Compare $C'(100)$ with the cost of producing the 101st item.

Solution

Part (a)

The marginal cost function is the derivative of the cost function.

$$\begin{aligned}\frac{dC}{dx} &= \frac{d}{dx}(920 + 2x - 0.02x^2 + 0.00007x^3)\\ &= 2(1) - 0.02(2x) + 0.00007(3x^2)\\ &= 2 - 0.04x + 0.00021x^2\end{aligned}$$

Part (b)

Plug in $x = 100$ to the formula in part (a) to get $C'(100)$.

$$C'(100) = 2 - 0.04(100) + 0.00021(100)^2 = 0.1$$

This is the rate that the cost increases with respect to the number of units produced, and it serves as an estimate for the cost to produce the 101st unit, namely \$0.10. It's only an estimate because $C'(100)$ is being interpreted as the slope of the secant line over the interval $100 \leq x \leq 101$ rather than as the slope of the tangent line at $x = 100$.

Part (c)

The actual cost of the 101st item is

$$\begin{aligned}C(101) - C(100) &= [920 + 2(101) - 0.02(101)^2 + 0.00007(101)^3]\\ &\qquad - [920 + 2(100) - 0.02(100)^2 + 0.00007(100)^3]\\ &= (990.10107) - (900)\\ &= 0.10107.\end{aligned}$$

Use the percent difference formula to find how much higher this number is than the estimate.

$$\frac{0.10107 - 0.10}{0.10} \times 100\% = 1.07\%$$